

21. $x = 2\sin t$ highest point
 $y = \cos 2t$ $1.5 \leq t \leq 4.5$

$\frac{dy}{dt} = -2\sin 2t = 0$
 $2t = \frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}$
 ~~$t = 0, \frac{\pi}{2}, \pi$~~ ~~$t = 0, \frac{\pi}{2}, \pi$~~
 ~~$x = 0, y = 1$~~ ~~$x = 0, y = 1$~~
 $t = \pi$ $x = 0, y = 1$

Feb 8-9:25 AM

10.2 Vectors $\hat{u} = \langle -2, 1 \rangle$ $\hat{v} = \langle 5, 12 \rangle$ or (r, θ)

What is the magnitude of the vector $\langle 5, 12 \rangle$?

$\hat{u} + \hat{v} = \hat{w}$
 $\hat{w} = \langle 3, 13 \rangle$

$r = \sqrt{x^2 + y^2}$
 $\theta = \tan^{-1}(\frac{y}{x})$
 $x = r \cos \theta$
 $y = r \sin \theta$

$\text{mag } |\hat{v}| = 13 = r$
 $\theta = \tan^{-1} \frac{12}{5} = 1.17$ or 67.38°

Feb 20-5:58 PM

Vector valued functions speed = $|\hat{v}(t)|$

A particle moves in the plane so that its position at any time $t \geq 0$ is given by $(\sin t, t^2/2)$.

(a) Find the position vector $\hat{r}(t) = (\sin t, \frac{t^2}{2})$
 (b) Find the velocity vector $\hat{v}(t) = (\cos t, t)$
 (c) Find the acceleration vector $\hat{a}(t) = (-\sin t, 1)$
 (d) Determine the path of the particle
 (e) Describe the position and motion of the particle at time $t = 4$
 (f) Find the velocity and the speed at time $t = 4$

$\hat{r}(1) = (0.84, 0.5)$
 $\hat{r}(4) = (0.76, 8)$
 $\hat{v}(4) = (-0.76, 4)$
 $\hat{a}(4) = (0.76, 1)$
 $\text{speed}(4) = \sqrt{(-0.76)^2 + 4^2} = 4.05$

Feb 20-6:06 PM

Displacement and Distance Traveled

A particle moves in the plane with velocity vector $v(t) = (t - 3\pi \cos \pi t, 2t - \pi \sin \pi t)$.At $t = 0$ the particle is at the point $(1, 5)$ $\hat{v}(t) = (\frac{dx}{dt}, \frac{dy}{dt})$ (a) Find the position of the particle at $t = 4$.(b) What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

$$(a) \hat{r}(t) = (\frac{t^2}{2} - 3\sin \pi t + C_1, t^2 \cos \pi t + C_2)$$

$$1 = 0 - 3\sin 0 + C_1 \quad 5 = 0^2 \cos 0 + C_2$$

$$1 = C_1 \quad C_2 = 5$$

$$\hat{r}(4) = (\frac{4^2}{2} - 3\sin 4\pi + 1, 4^2 \cos 4\pi + 5)$$

$$\hat{r}(4) = (9, 21)$$

$$(b) \int_0^4 \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \text{arc length} = \text{distance}$$

$$\int_0^4 \sqrt{(t - 3\pi \cos \pi t)^2 + (2t - \pi \sin \pi t)^2} dt$$

$$= 33.5327$$

Feb 20-6:14 PM